RBSE BOARD
CLASS- X th

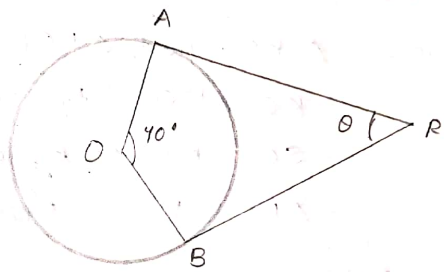
CIRCLE AND TANGENT [PREVIOUS YEAR 2015-19]

① If tangent RA and RB from a point R to a circle with center O are inclined to each other at an angle of θ and ∠AOB = 40°, then find the value of θ. [RBSE 2015]

Sol Consider,

Angle b/w Radius and tangent

$A = 90°$
$B = 90°$



from □OBRA,

$\angle AOB + \angle ARB = 180$
$40 + \angle ARB = 180$
$\angle ARB = 180 - 40 = 140$
$\angle ARB = 140$

② How many tangent can be constructed to any point on the circle of radius 4 cm? [RBSE 2015]

Sol. One point = One tangent

So, one tangent can be drawn for a given point.

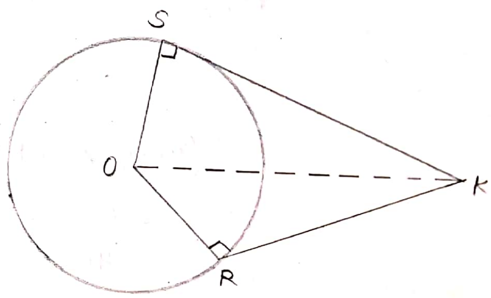
③ In the given figure, O is the center and two tangent KR, KS are drawn on the circle from a point K lying outside the circle. Prove that KR = KS.

Sol. Given, KR = KS (to prove) [RBSE 2015]

↓

This tangent are drawn to a circle with center O from an external point K.

Now, ΔKOS and ΔKOR



KO = KO (By common factor)

SO = RO (By radius of the circle)

$\angle 1 = \angle 2$ (Right angle (90))

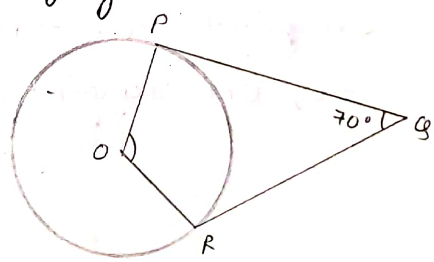
RHS

$\Delta KOS \cong \Delta KOR$

KR = KS (By CPCT)

④ In the given figure, O is the center of a circle and two tangents QP and QR are drawn on the circle from a point Q lying outside the circle. Find the value of angle POQR. [RBSE 2016]

Sol Consider,



Angle b/w Radius and tangent

$\angle P = 90°$

$\angle R = 90°$

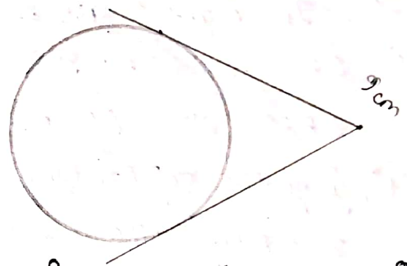
from $\triangle OPQR$,

$\angle POR + \angle PQR = 180$

$\angle POR + 70 = 180$

$\angle POR = 180 - 70$

$\angle POR = 110$



⑤ How many tangent can be drawn on the circle of radius 5 cm from a point lying outside the circle at distance 9 cm from the center. [RBSE 2016]

Sol Here from the diagram, it is very clear that the two tangent can be drawn from the point.

⑥ In the given, O is the center of a circle and two tangents CA, CB are drawn on the circle from a point C lying outside the circle. Prove that $\angle AOB$ and $\angle ACB$ are supplementary. [RBSE 2016]

Sol Consider the $\triangle OCA$ and $\triangle OBC$

$CA = CB$ (Tangent drawn from external points are equal)

$OA = OB$ (Radii of the circle)

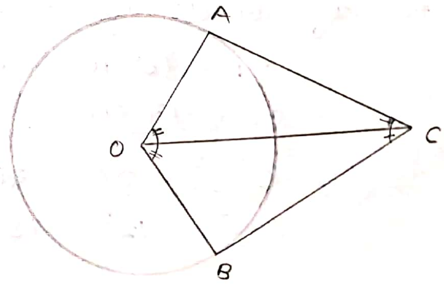
OC

Therefore $\triangle OCA \cong \triangle OBC$ (SSS cong. criteria)

Hence

$\angle OCA = \angle OCB$

$\angle AOC = \angle BOC$



Also, $\angle ACB = 2\angle OCA$ — ①

$\angle AOB = 2\angle(AOC)$ — ②

Also, In right angle $\triangle$ OAC, $\angle AOC + \angle OCA + \angle OAC = 180°$

[OAC = 90°] $\angle AOC + \angle OCA = 90°$

$\angle AOC = 90° - \angle OCA$ —(iii)

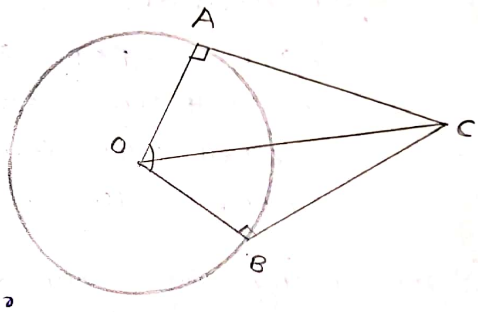
Multiplying the eqⁿ ③ by 2,

$2\angle AOC = 180° - 2\angle OCA$

By substituting ① and ② in eq above

$\angle AOB = 180° - \angle ACB \Rightarrow \angle AOB + \angle ACB = 180°$

$\times [\angle AOC + \angle OCA = 180°]$



HP.

⑦ From a point Q, the length of the tangent to a circle is 15 cm and the distance of Q P from the center of circle is 17 cm. then find the radius of the circle. [RBSE 2017]

Sol. Simplify the expression,

we have a tangent of 15 cm and the distance of Q from the centre of circle is 17 cm.

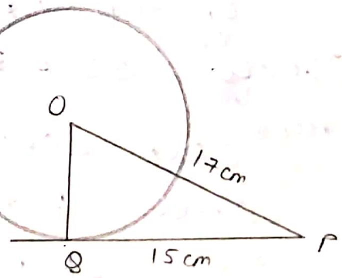
QP = 15 cm then OQ = ?

OP = 17 cm

$OQ^2 = OP^2 - QP^2$

$OQ^2 = 17^2 - 15^2 \Rightarrow OQ^2 = 289 - 225 = 64$

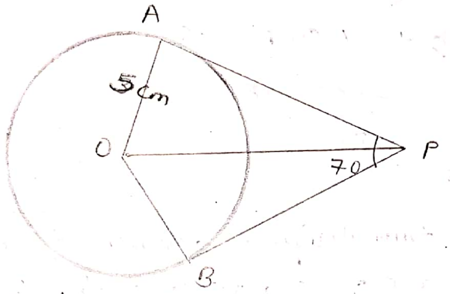
OQ = 8 cm



Hence, Radius = 8 cm.

⑧ Draw a pair of tangent to a circle of radius 5 cm which are inclined to each at an angle of 70°. [RBSE 2017]

Sol Drawn the figure



⑨ Prove that the angle b/w the two tangents drawn from a external points to a circle is supplementary to the angle subtended by the line-segment joining the points of the contact the centre. [RBSE 2017]

Sol Prove the expression,

Let us consider PT and QT are the tangents line from the external point T. which touches the circle at P and Q respectively.

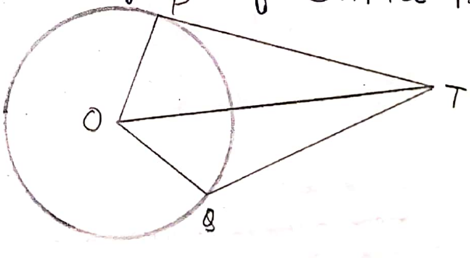
O is the center of this circle OP and OQ are the radius of this circle.

OP ⊥ PT [angle b/w radius and tangent at point of contact is 90°]

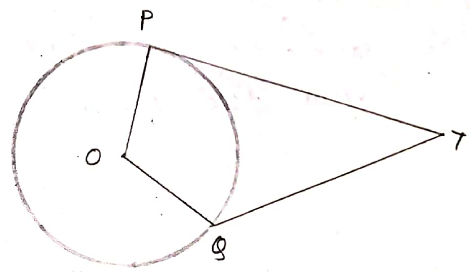
$\angle OPT = 90°$ — (i)

OQ ⊥ TQ ["]

$\angle OQT = 90°$ — (ii)



Now, in quadrilateral POQT



$\angle OPT + \angle PTQ + \angle OQT + \angle QOP = 360°$

$90° + 90° + \angle PTQ + \angle QOP = 360°$

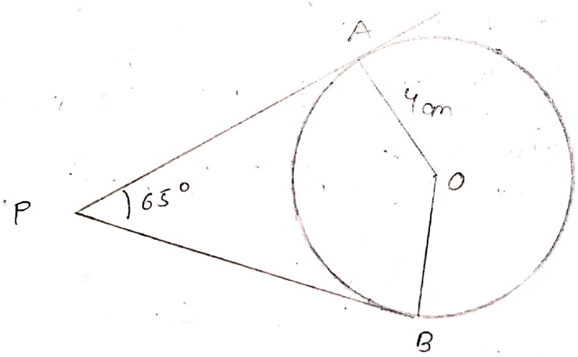
$\angle PTQ + \angle QOP = 360° - 180°$

$= 180°$

Since the angle b/w the two tangent drawn from an external point to a circle is supplementary to the angle substended by the line-segment.

(10) Draw two tangents PA and PB from an external point P, to a circle of radius 4 cm, where angle b/w PA and PB is 65°. [RBSE 2018]

Sol.



(11) A circle with centre 'O' touches all the four sides of a quadrilateral ABCD internally in such a way that it divide AB in 3 : 1 and AB = 12 cm then find the radius of the circle where OA = 15 cm . [RBSE 2019]

Sol let $OA = 15$ cm

say, the circle touches AB at P

AB is divided by P in 3 : 1

$AP = (3/(3+1)) \times AB$

$= \frac{3}{4} AB = \frac{3}{4} \times 12$

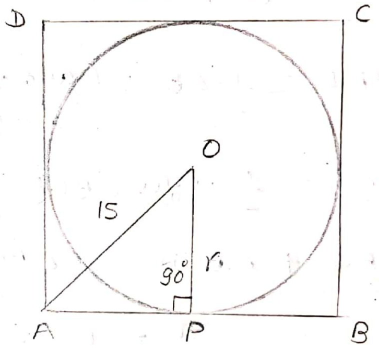
$AP = 9$

$\frac{AP}{BP} = \frac{3}{1}$

$AP = 3BP$

$AB = AP + BP$

$AB = \frac{4}{3} AP$



So, $OA^2 = OP^2 + AP^2$

$(15)^2 = OP^2 + 9^2$

$OP^2 = 225 - 81$

$OP = 12$ cm

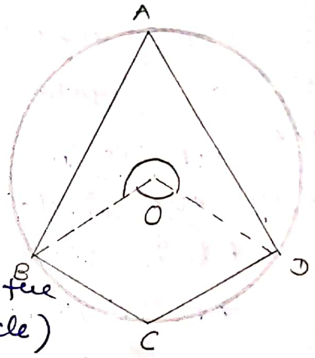
(12) Prove that the opposite angle of cyclic quadrilateral are supplementary or sum is 180°.

OR [RBSE 2019]

Prove that if a chord is drawn from a point of contact of the tangent of the circle then the angle made by this chord with the tangent is equal to the respective alternate angle made by segments with the chord.

Sol O is the centre of the circle

ABCD is a cyclic quadrilateral



To prove : $\angle BAD + \angle BCD = 180°$ and

$\angle ABC + \angle ADC = 180°$

Construction : Join OB and OD

Proof :

(i) $\angle BAD = \frac{1}{2} \angle BOD$ (angle subst. by an arc at the centre is double the angle on the circle)

(ii) $\angle BCD = \frac{1}{2}$ reflex $\angle BOD$

(iii) $\angle BAD + \angle BCD = \frac{1}{2} \angle BOD + \frac{1}{2}$ reflex $\angle BOD$

$= \frac{1}{2} (\angle BOD + \frac{1}{2}$ reflex $\angle BOD)$

$= \frac{1}{2} (360°)$ [complete angle at the centre]

$\angle BAD + \angle BCD = 180°$

Similarly $\angle ABC + \angle ADC = 180°$.

OR

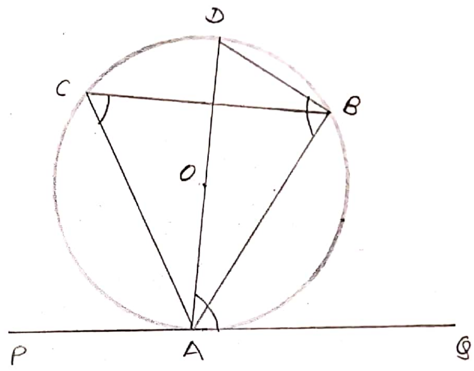
Here , $\angle ACB = \angle ADB$ (circle of segments are equal) — ①

Now,

$\angle ABD = 90^\circ$ [make an angle of semicircle]

$\angle DAQ = 90^\circ$ [DA is $\perp$ to PQ]

In an $\triangle ABD$



$\angle ABD + \angle BAD + \angle ADB = 180^\circ$

$90^\circ + \angle BAD + \angle ADB = 180^\circ$

$\angle BAD + \angle ADB = 180^\circ - 90^\circ$

$\angle BAD + \angle ADB = 90^\circ$

But $\angle DAQ = 90^\circ$

So, $\angle BAD + \angle ADB = \angle DAQ$

Also, $\angle DAQ = \angle BAD + \angle BAQ$

Therefore

$\angle BAD + \angle ADB = \angle BAD + \angle BAQ$

$\angle ADB = \angle BAQ$

from eq^n 1 ($\angle ADB = \angle ACB$)

Hence $\angle ACB = \angle BAQ$ HP.