SHARMA TUTION CLASSES Let's Rule it

RBSE BOARD

CLASS - XII[th]

DETERMINANTS [PREVIOUS YEAR 2015-19]

① Prove that $\begin{vmatrix} a+b+2c & c & c \\ a & b+c+2a & a \\ b & b & c+a+2b \end{vmatrix} = 2(a+b+c)^2$ [RBSE 2015]

Sol. Simplify the matrix expression

LHS. $\begin{vmatrix} a+b+2c & c & c \\ a & b+c+2a & a \\ b & b & c+a+2b \end{vmatrix}$

$R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} 2(a+b+c) & 2(a+b+c) & 2(a+b+c) \\ a & b+c+2a & a \\ b & b & c+a+2b \end{vmatrix}$$

$$= 2(a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ a & b+c+2a & a \\ b & b & c+a+2b \end{vmatrix}$$

$C_2 \to C_2 - C_1 \qquad C_3 \to C_3 - C_1$

$$= 2(a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ a & a+b+c & 0 \\ b & 0 & a+b+c \end{vmatrix}$$

$$= 2(a+b+c)^2 \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & 0 & 1 \end{vmatrix}$$

$$= 2(a+b+c)^2 \left[1(1-0) + 0 + 0 \right]$$

$$= 2(a+b+c)^2 \quad \text{H.P.}$$

(2) Prove that $\begin{vmatrix} a & a^2 & b+c \\ b & b^2 & c+a \\ c & c^2 & a+b \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$ [RBSE 2016]

Sol. Simplify the expression, $\begin{vmatrix} a & a^2 & b+c \\ b & b^2 & c+a \\ c & c^2 & a+b \end{vmatrix}$

$C_1 \to C_1 + C_3$

$$= \begin{vmatrix} a+b+c & a^2 & b+c \\ b+c+a & b^2 & c+a \\ c+a+b & c^2 & a+b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & a^2 & b+c \\ 1 & b^2 & c+a \\ 1 & c^2 & a+b \end{vmatrix}$$

$R_1 \to R_1 - R_3$

$$= (a+b+c)\begin{vmatrix} 0 & a^2-c^2 & c-a \\ 0 & b^2-c^2 & c-b \\ 1 & c^2 & a+b \end{vmatrix}$$

Now, solving

$$= (a+b+c)\left[1(a^2-c^2)(c-b) - (c-a)(b^2-c^2)\right]$$

$$= (a+b+c)\left[(a-c)(a+c)(c-b) - (c-a)(b-c)(b+c)\right]$$

$$= (a+b+c)(b-c)(c-a)\left[(a+c)-(b+c)\right]$$

$$= (a+b+c)(a-b)(b-c)(c-a).$$ Hp.

③ Show that $\begin{vmatrix} a & a^2 & 1+pa^3 \\ b & b^2 & 1+pb^3 \\ c & c^2 & 1+pc^3 \end{vmatrix} = (1+pabc)(a-b)(b-c)(c-a)$ [RBSE 2017]

Sol.

Simplify the expression,

$$\begin{vmatrix} a & a^2 & 1+pa^3 \\ b & b^2 & 1+pb^3 \\ c & c^2 & 1+pc^3 \end{vmatrix} = \begin{vmatrix} a & a^2 & pa^3 \\ b & b^2 & pb^3 \\ c & c^2 & pc^3 \end{vmatrix} + \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + abcp \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$= (1 + abcp) \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

$R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$= (abcp+1) \begin{vmatrix} a-b & a^2-b^2 & \cancel{1}0 \\ b-c & b^2-c^2 & \cancel{1}0 \\ c & c^2 & 1 \end{vmatrix}$$

Solving it,

$$= (1 + abcp)\left[(a-b)(b^2-c^2) - (b-c)(a^2-b^2) \right]$$

$$= (1 + abcp) \left[(a-b)(b-c)\left[(b+c) - (a+b)\right]\right]$$

$$= (1+abcp)\left[(a-b)(b-c)(a+b)\right]$$

HP.

④ If $A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$ then find A^{-1}. [RBSE 2018]

Sol Here, $A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$

we know that $A^{-1} = \frac{1}{|A|} (\text{adj } A)$

$|A| = 6 - 4 = 2$

$\text{adj } A = \begin{bmatrix} 2 & -4 \\ -1 & 3 \end{bmatrix}$

So, $A^{-1} = \frac{1}{2} \begin{bmatrix} 2 & -4 \\ -1 & 3 \end{bmatrix}$

⑤ Prove that $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(x-4)^2$ (RBSE 2018)

Sol. taking LHS $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} 5x+4 & 5x+4 & 5x+4 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

$$= (5x+4)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

$C_1 \to C_1 - C_2$; $C_2 \to C_2 - C_3$

$$= (5x+4)\begin{vmatrix} 0 & 0 & 1 \\ x-4 & 4-x & 2x \\ 0 & x-4 & x+4 \end{vmatrix}$$

solving determinant

$= (5x+4)\left[1\,(4-x)(x+4) - 2x\,(x-4)\right]$

$= (5x+4)\left[4x + 16 - x^2 - 4x - 2x^2 + 4\right]^{\times}$

$= (5x+4)\left[(x-4)^2\right]$

$= (x-4)^2(5x+4)$ HP.

⑥ Solve the following eq^n by using Cramer's rule. $5x - 4y = 7$ [RBSE 2018]
$x + 3y = 9$

Sol. Given,

$$5x - 4y = 7 \quad -(i)$$
$$x + 3y = 9 \quad -(ii)$$

using Cramer's Rule

$$\Delta = \begin{vmatrix} 5 & -4 \\ 1 & 3 \end{vmatrix} = [(5\times3) - (-4\times1)] = 15 + 4 = 19$$

$$\Delta_1 = \begin{vmatrix} 7 & -4 \\ 9 & 3 \end{vmatrix} = [(7\times3) - (-4\times9)] = 21 + 36 = 57$$

$$\Delta_2 = \begin{vmatrix} 5 & 7 \\ 1 & 9 \end{vmatrix} = [(5\times9) - (7\times1)] = 45 - 7 = 38$$

So, $x = \dfrac{\Delta_1}{\Delta} = \dfrac{57}{19} = 3$, $y = \dfrac{\Delta_1}{\Delta} = \dfrac{38}{19} = 2$

⑦ If matrix $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then find A^{-1}. [RBSE 2019]

Sol Given, $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

and $A^{-1} = \dfrac{1}{|A|}(\text{adj} A)$

$|A| = \cos^2\theta + \sin^2\theta = 1$

$$\text{adj}A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

8. If $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ 2 & 5 \end{bmatrix}$, then prove that $(AB)^T = B^T A^T$. [RBSE 2019]

Sol. Here $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ 2 & 5 \end{bmatrix}$

LHS.

$$AB = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 2+6 & -2+15 \\ -1+8 & 1+20 \end{bmatrix} = \begin{bmatrix} 8 & 13 \\ 7 & 21 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 8 & 7 \\ 13 & 21 \end{bmatrix}$$

Now, RHS. $$B^T A^T = \begin{bmatrix} 1 & 2 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2+6 & -1+8 \\ -2+15 & 1+20 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} 8 & 7 \\ 13 & 21 \end{bmatrix}$$ HP.

⑨ Prove that $\begin{vmatrix} 1+a & b & c \\ a & 1+b & c \\ a & b & 1+c \end{vmatrix} = (1+a+b+c)$ [RBSE 2019]

Sol. LHS. $= \begin{vmatrix} 1+a & b & c \\ a & 1+b & c \\ a & b & 1+c \end{vmatrix}$

$C_1 \to C_1 + C_2 + C_3$ $\begin{vmatrix} 1+a+b+c & b & c \\ 1+a+b+c & 1+b & c \\ 1+a+b+c & b & 1+c \end{vmatrix} = (1+a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & 1+b & c \\ 1 & b & 1+c \end{vmatrix}$

$R_2 \to R_2 - R_1$; $R_3 \to R_3 - R_1$

So, $= (1+a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$= (1+a+b+c)[1(1-0)]$

$= (1+a+b+c)$ HP.