RBSE BOARD

CLASS - XIIth

SHARMA TUTION CLASSES Let's Rule it

MATRICES [PREVIOUS YEAR 2015-19]

① If $2A + B = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -5 \\ 0 & 2 \end{bmatrix}$ then find A. [RBSE 2015]

Sol. Given that,

$$2A + B = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & -5 \\ 0 & 2 \end{bmatrix}$$

So,

$$2A = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} -1 & -5 \\ 0 & 2 \end{bmatrix} \Rightarrow 2A = \begin{bmatrix} 4 & 4 \\ 2 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$$

② If $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ then find AB [RBSE 2015]

Sol. Here, $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Then, $AB = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 1 \times 1 + 2 \times 2 + 3 \times 3$

$= 14$

(3) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ and $A^2 - 4A = KI_3$ find the value of k. [RBSE 2015]

Sol. We have, $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

we know that, $A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+4 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

Now, $A^2 - 4A - KI_3 = 0$

$$\begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - K\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 1 & 8 \\ 8 & 8 & 1 \end{bmatrix} - \begin{bmatrix} K & 0 & 0 \\ 0 & K & 0 \\ 0 & 0 & k \end{bmatrix} = 0$$

$$\begin{bmatrix} 9-4 & 8-8 & 8-8 \\ 8-8 & 9-4 & 8-8 \\ 8-8 & 8-8 & 9-4 \end{bmatrix} - \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} = 0$$

$\Rightarrow \quad 5-k = 0$

$k = 5$

Hence, the value of $k = 5$.

④ Construct a 2×2 matrix $A = [a_{ij}]$, whose elements are given by $a_{ij} = |-5\hat{i}+2\hat{j}|$.

[RBSE 2016]

Sol. Here, $a_{ij} = |-5\hat{i}+2\hat{j}|$

The matrix A of order 2×2 is given by $\begin{bmatrix} a_{11} & a_{12} \\ a_{13} & a_{14} \end{bmatrix}$

$a_{11} = |-5+2| = 3$

$a_{12} = |-5+4| = 1$

$a_{21} = |-10+2| = 8$

$a_{22} = |-10+4| = 6$

thus $A = \begin{bmatrix} 3 & 1 \\ 8 & 6 \end{bmatrix}$

⑤ If $[x-3]\begin{bmatrix} 2x \\ 6 \end{bmatrix} = 0$, then find the value of x. [RBSE 2016]

Sol.

$$[x \;\; -3]\begin{bmatrix} 2x \\ 6 \end{bmatrix} = 0$$

$$2x^2 - 18 = 0$$

$$2x^2 = 18 \Rightarrow x^2 = 9$$

$$x = \pm 3.$$

⑥ Express the matrix $A = \begin{bmatrix} 2 & -4 & -2 \\ -1 & 4 & 3 \\ 1 & -3 & 2 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix. [RBSE 2016]

Sol. We know,

Sum can be represented as $\left[\dfrac{A + A^T}{2}\right] + \left[\dfrac{A - A^T}{2}\right]$

$$= \frac{1}{2}\left(\begin{bmatrix} 2 & -4 & -2 \\ -1 & 4 & 3 \\ 1 & -3 & 2 \end{bmatrix} + \begin{bmatrix} 2 & -1 & 1 \\ -4 & 4 & -3 \\ -2 & 3 & 2 \end{bmatrix}\right) + \frac{1}{2}\left(\begin{bmatrix} 2 & -4 & -2 \\ -1 & 4 & 3 \\ 1 & -3 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -1 & 1 \\ -4 & 4 & -3 \\ -2 & 3 & 2 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 4 & -5 & -1 \\ -5 & 8 & 0 \\ -1 & 0 & 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -3 & -3 \\ 3 & 0 & 6 \\ 3 & -6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -5/2 & -1/2 \\ -5/2 & 4 & 0 \\ -1/2 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -3/2 & -3/2 \\ 3/2 & 0 & 3 \\ 3/2 & -3 & 0 \end{bmatrix}$$

(7) If $A = \begin{bmatrix} 2 & 4 \\ -3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$, then find $2A - B$. [RBSE 2017.]

Sol. Here, $A = \begin{bmatrix} 2 & 4 \\ -3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$

$$2A - B = 2\begin{bmatrix} 2 & 4 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 8 \\ -6 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 5 \\ -4 & -1 \end{bmatrix}$$

(8) If $A = [2\ -4\ 3]$ and $B = \begin{bmatrix} 2 \\ -4 \\ 8 \end{bmatrix}$, then find $(AB)'$ [RBSE 2017]

Sol. Here, $A = [2\ \ -4\ \ 3]$ $B = \begin{bmatrix} 2 \\ -4 \\ 8 \end{bmatrix}$

So,

$$AB = [2\ \ -4\ \ 3]\begin{bmatrix} 2 \\ -4 \\ 8 \end{bmatrix}$$

$$AB = [4 + 16 + 24] \Rightarrow AB = [44]$$

$$AB' = [44]$$

(9) If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then prove that $A^2 - 5A + 7I_2 = 0$, where I_2 is the identity matrix of order 2. [RBSE 2017]

Sol. Here, $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$A^2 - 5A + 7I_2 = 0$, So, $A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$A^2 - 5A + 7I_2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(10) Find A, if $2A - \begin{bmatrix} 3 & -1 \\ 1 & 2 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 3 & 2 \\ 0 & 1 \end{bmatrix}$ [RBSE 2018]

Sol. Here,

$$2A - \begin{bmatrix} 3 & -1 \\ 1 & 2 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 3 & 2 \\ 0 & 1 \end{bmatrix}$$

$$2A = \begin{bmatrix} 1 & 5 \\ 3 & 2 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ 1 & 2 \\ 0 & 8\,1 \end{bmatrix}$$

$$2A = \begin{bmatrix} 4 & 4 \\ 4 & 4 \\ 0 & 6 \end{bmatrix}$$

$$A = \frac{1}{2}\begin{bmatrix} 4 & 4 \\ 4 & 4 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 4/2 & 4/2 \\ 4/2 & 4/2 \\ 0 & 6/2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \\ 0 & 3 \end{bmatrix}$$

(11) If $A = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & -2 \\ 1 & 2 \end{bmatrix}$ then find $2A^2 - 3B$. [RBSE 2018]

Sol. Here, A and B is given,

So, $A^2 = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1-4 & -2-6 \\ 2+6 & -4+9 \end{bmatrix} = \begin{bmatrix} -3 & -8 \\ 8 & 5 \end{bmatrix}$

$$3B = \begin{bmatrix} -5 & -2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -15 & -6 \\ 3 & 6 \end{bmatrix}$$

Now, $2A^2 - 3B = 2\begin{bmatrix} -3 & -8 \\ 8 & 5 \end{bmatrix} - 3\begin{bmatrix} -15 & -6 \\ 3 & 6 \end{bmatrix}$

$$= \begin{bmatrix} -6 & -16 \\ 16 & 10 \end{bmatrix} - \begin{bmatrix} -15 & -6 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 9 & -10 \\ 13 & 4 \end{bmatrix}$$

(12) If $\begin{bmatrix} a+b & 4 \\ -3 & ab \end{bmatrix} = \begin{bmatrix} 6 & 4 \\ -3 & 8 \end{bmatrix}$, then find the value of a and b. [RBSE 2019]

Sol. Since the matrices are equal,

so, $a+b = 6$ —(i)

$ab = 8 \Rightarrow a = \frac{8}{b}$ —(ii)

Put (ii) in eq^n (i)

$\frac{8}{b} + b = 6 \Rightarrow 8 + b^2 = 6b$

$\Rightarrow b^2 - 6b + 8 = 0$

$\Rightarrow b^2 - 4b - 2b + 8 = 0$

$b(b-4) - 2(b-4) = 0$

$b = 2, \quad a = 4$

$b = 4, \quad a = 2$

(13) Solve system of linear eq^n, using matrix method. → $2x + 3y + 3z = 5$, $x - 2y + z = -4$, $3x - y - 2z = 3$ [RBSE 2019]

Sol. Here, the system of eq^n is

$2x + 3y + 3z = 5$

$x - 2y + z = -4$

$3x - y - 2z = 3$

writing eq^n as $AX = B$

$$\begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

Hence, $A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$

Calculating $|A| = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}$

$$= 2 \begin{vmatrix} -2 & 1 \\ -1 & -2 \end{vmatrix} - 3 \begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix} + 3 \begin{vmatrix} 1 & -2 \\ 3 & -1 \end{vmatrix}$$

$$= 2(4+1) - 3(-2-3) + 3(-1+6)$$

$$= 2(5) - 3(-5) + 3(5) = 10 + 15 + 15$$

$$= 40$$

Since, $|A| = 0$

$\therefore$ The system of eq is consistent and has a unique solution

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Now, $AX = B$

$X = A^{-1}B$

Calculating A^{-1} $\quad A^{-1} = \frac{1}{|A|} \operatorname{adj}(A)$

$$\operatorname{adj} A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}' = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{23} & A_{22} & A_{33} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}$$

Now, we can now find $\operatorname{adj} A = \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$

Also, $X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix} \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 5(5) + 3(-4) + 9(3) \\ 5(5) + (-13)(-4) + 1(3) \\ 5(5) + 11(-4) + 7(-3) \end{bmatrix}$$

$$= \frac{1}{40} \begin{bmatrix} 25 - 12 + 27 \\ 25 + 52 + 3 \\ 25 - 44 - 21 \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$, they $x = 1$, $y = 2$, $z = -1$.