RBSE BOARD

CLASS-X^th

SHARMA TUTION CLASSES ▶ Let's Rule it

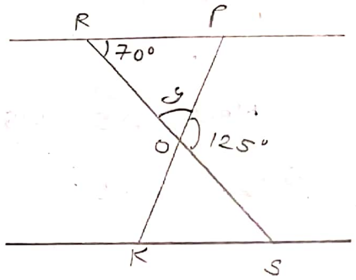
SIMILARITY [PREVIOUS YEAR 2015-19]

① In the figure, $\triangle OPR \sim \triangle OSK$, $\angle POS = 125^\circ$ and $\angle PRO = 70^\circ$. Find the value of $\angle OKS$ and $\angle ROP$. [RBSE 2015]

Sol Given that,

$\triangle OPR \sim \triangle OSK$

$\angle POS = 125^\circ$ and $\angle PRO = 70^\circ$.



Now, the linear pair

$y + 125 = 180$

$y = 180 - 125$

$y = 55$

Now, angle sum properties says,

sum of all angles of $\triangle$ is 180°

So, $\angle P + 70^\circ + y = 180$

$\angle P = 180 - 70 - 55$

$\angle P = 55$

Therefore, $\angle P = \angle OKS$ $(\because \triangle OPR \sim \triangle OBK)$

$\therefore \angle OKS = 55$.

② O is any point inside rectangle ABCD. Prove that $OB^2 + OD^2 = OA^2 + OC^2$. [RBSE 2015]

Sol

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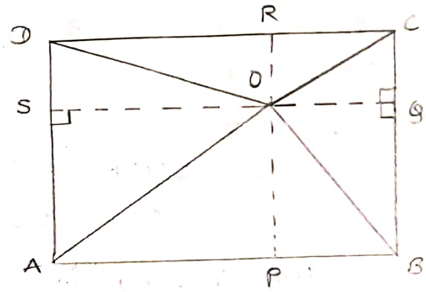
Sol. Consider,

the point O construct ⊥ OP, OR, OS and OS to the sides of the △

to prove

$$OB^2 + OD^2 = OA^2 + OC^2.$$



taking RHS

$$OA^2 + OC^2 = (AS^2 + OS^2) + (OQ^2 + QC^2)$$

Here $AS = BQ$ and $QC = SD$

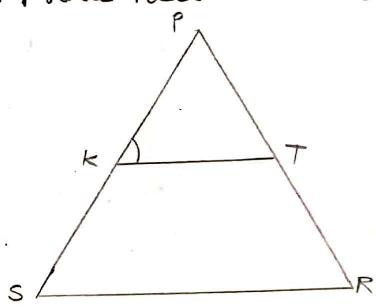
So, $OA^2 + OC^2 = BQ^2 + OS^2 + OQ^2 + SD^2$

$$= BQ^2 + OQ^2 + OS^2 + SD^2$$

$$OA^2 + OC^2 = OB^2 + OD^2$$

HP.

③ In the given figure $\frac{PK}{KS} = \frac{PT}{TR}$ and $\angle PKT = \angle PRS$. Prove that $\triangle PSR$ is an isosceles △. [RBSE 2015]



Sol. Given that, $\frac{PK}{KS} = \frac{PT}{TR}$ and $\angle PKT = \angle PRS$

Now, $\triangle PSR$, $\frac{PK}{KS} = \frac{PT}{TR}$, then $KT \parallel SR$

$$\angle PKT = \angle PSR \quad -(i)$$

Also $\angle PKT = \angle PRS$ (given

So, $\angle PSR = \angle PRS$, So $\triangle PSR$ is a Isosceles △

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(4) The area of two similar $\triangle$ are in ratio 16:81. Find the ratio of its sides.

[RBSE 2016]

Sol Given that, $\triangle ABC \sim \triangle PQR$

$$\frac{ar(ABC)}{ar(PQR)} = \left(\frac{AB}{PQ}\right)^2$$

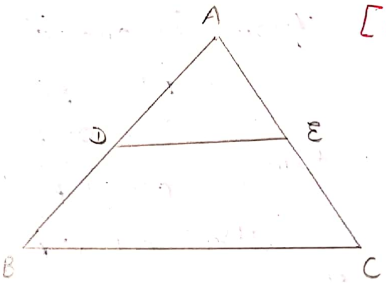
[ratio of areas of two similar $\triangle$ is equal to ratio of squares of their corresponding sides]

$$\frac{16}{81} = \left(\frac{AB}{PQ}\right)^2 \Rightarrow \frac{AB}{PQ} = \frac{4}{9}$$

(5) In the given figure, ABC is a $\triangle$. If $AC/AE = AB/AD$, then prove that $BC \parallel DE$

[RBSE 2016]

Sol. Given that,



$$\frac{AC}{AE} = \frac{AB}{AD}$$

construction - draw DE

Proof: $\frac{AC}{AE} = \frac{AB}{AD}$

Subtract 1 both side.

$$\frac{AC}{AE} - 1 = \frac{AB}{AD} - 1 \Rightarrow \frac{AC - AE}{AE} = \frac{AB - AD}{AD}$$

$$\Rightarrow \frac{CE}{AE} = \frac{DB}{AD} \Rightarrow \frac{AE}{EC} = \frac{AD}{DB}$$

$\therefore$ $DE \parallel BC$ (By opposite of BPT)

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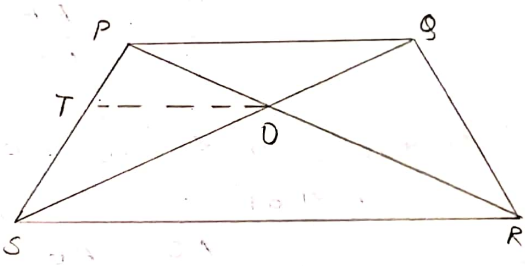
OR

The diagonals of a quadrilateral PQRS intersect each other at a point O such that $\frac{PO}{OR} = \frac{OQ}{OS}$. Show that PQRS is a trapezium. [RBSE 2016]

Sol (In $\triangle PQR$, OE ∥ AB) Constr.: From O draw line OT ∥ PQ

By using BPT, in $\triangle PQS$



$$\frac{PT}{TS} = \frac{OQ}{OS} \quad -(i)$$

Also, it is given $\frac{PO}{OR} = \frac{OQ}{OS} \quad -(ii)$

from (i) and (ii)

$$\frac{PT}{TS} = \frac{PO}{OR}$$

thus, OT ∥ RS

also OT ∥ PQ

thus PQ ∥ RS, Hence PQRS is a trapezium.

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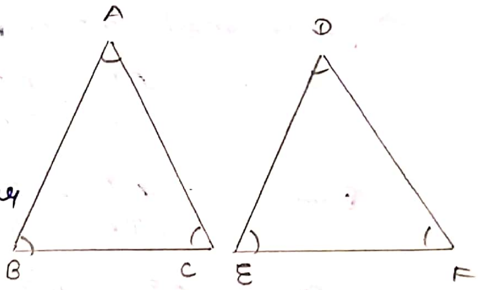
⑥ If the ratio of corresponding medians of two similar triangle are 9:16, then find the areas ratio. [RBSE 2017]

Sol simplify the expression,

we know that

ΔABC and ΔDEF



$$\left.\begin{array}{l} \angle A = \angle D \\ \angle B = \angle E \\ \angle C = \angle F \end{array}\right\} \text{ as } \Delta \text{ are similar}$$

also $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{DF}$

Then, $\frac{\text{area of } \Delta ABC}{\text{area of } \Delta ADF} = \frac{(AB)^2}{(DE)^2} = \frac{(9)^2}{(16)^2} = \frac{81}{256}$

Ratio will be ar(ABC) : ar(ADF) = 81 : 256

⑦ Prove that a line drawn through the mid point of one side of a triangle parallel to the second side bisect the third side.

Sol Given: A line l intersect lines AB and AC of triangle ABC at D and E respectively such that

$$\frac{AD}{DB} = \frac{AE}{BC}$$

to prove: $l \parallel BC$ i.e $DE \parallel BC$

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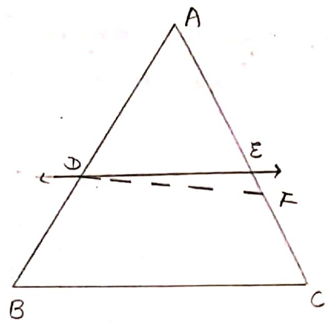
Proof: Let us suppose that DE is not parallel to BC. Then show other line is parallel to BC.

Suppose that DF ∥ BC



$\because \quad \frac{AD}{DB} = \frac{AF}{FC}$ (by BPT) — (1)

But, $\frac{AD}{DB} = \frac{AE}{EC}$ (Given) . — (2)

So, $\frac{A}{FC} = \frac{AE}{EC}$ (from (1) and (2))

Adding (1) to both sides

$\frac{AF}{FC} + 1 = \frac{AE}{EC} + 1 \Rightarrow \frac{AF + FC}{FC} = \frac{AE + EC}{EC}$

or $\frac{AC}{FC} = \frac{AC}{EC}$

Thus $FC = EC$ Hp.

OR

PQRS is a trapezium in which PQ ∥ RS and its diagonals intersects each at the point O. Prove that $\frac{PO}{QO} = \frac{RO}{SO}$ [RBSE 2017]

Sol.

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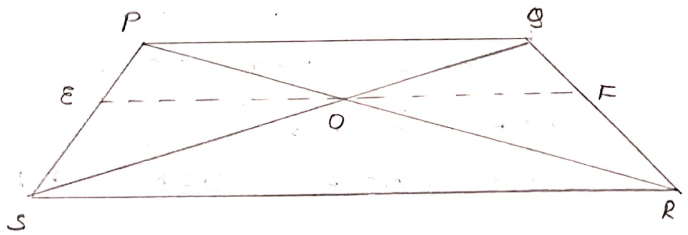
To prove: $\frac{PO}{QO} = \frac{RO}{SO}$



Construction: $EOF \parallel PQ$

Proof: In $\triangle PSR$

$EO \parallel SR$

By using the result of B.P.T

$\frac{PE}{ES} = \frac{PO}{RO}$ — (i)

In $\triangle PQR$, $EO \parallel PQ$

By using the result BPT

$\frac{PE}{ES} = \frac{QO}{SO}$ — (ii)

By (i) and (ii)

$\frac{PO}{QO} = \frac{RO}{SO}$ H.P.

(3) In given figure if $OP \cdot OQ = OR \cdot OS$, then show that $\angle OPS = \angle ORQ$, and $\angle OQR = \angle OSP$.

Sol. Prove the expression, $OP \cdot OQ = OR \cdot OS$

$\frac{OP}{OQ} = \frac{OS}{OQ}$

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And

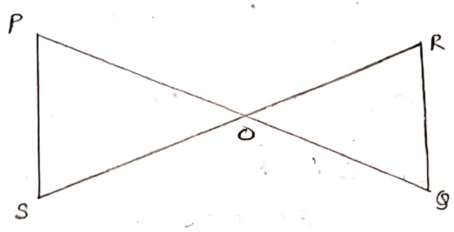
$\angle POS = \angle ROQ$

then

$\triangle POS \sim \triangle ROQ$ [SSA]

$\angle OPS = \angle ORQ$

and $\angle OSP = \angle OQR$ H P.

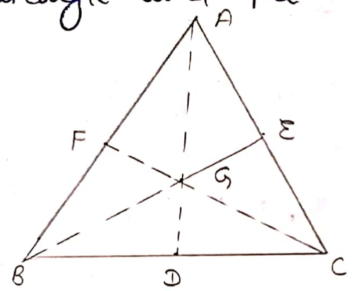


(9) In a $\triangle ABC$, the median AD, BE, and CF pass through the point G. If AD = 9 cm. GE = 4.2 cm, GC = 6 cm, then find the values of length of AG, BE and FG. [RBSE 2018]

Sol. Given that in $\triangle ABC$, median AD, BE and CF intersect at a point G.

GE = 4.2 cm, AD = 9 cm, GC = 6 cm.

As, we know that if the median in the triangle intersect at a point then point is the centroid of the triangle and the centroid divides the median in 2 : 1 ratio.



Here G is the centroid of $\triangle ABC$

So, AG : GD = 2 : 1

BG : GE = 2 : 1

CG : GF = 2 : 1

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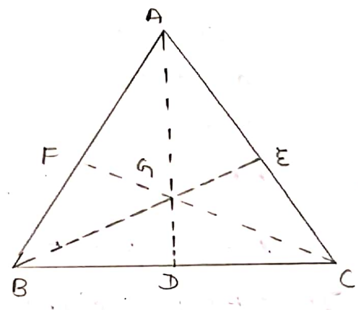
Now, $AD = AG + GD$

$\frac{AG}{GD} = \frac{2}{1} \Rightarrow AG = 2GD \quad \text{— (i)}$



$AD = AG + GD$

$AG = 2GD$

$AD = 2GD + GD = 3GD$

$AD = 9 \Rightarrow \frac{9}{3} = GD \Rightarrow GD = 3$

So, $AG = 2 \times GD$

$= 2 \times 3 = 6$ cm.

Again, $\frac{CG}{GF} = \frac{2}{1}$

$\Rightarrow FG = \frac{CG}{2} \Rightarrow CF = FG + GC$

$\Rightarrow CF = FG + 2FG$

Also,

$\Rightarrow CF = 3FG$

$FG = \frac{6}{2} = 3 \quad \Rightarrow CF = 3 \times 3 = 9$

Now, $\frac{BG}{GE} = \frac{2}{1} \Rightarrow BG = 2GE$

$BE = BG + GE$

$= 2GE + GE$

$= 3GE = 3 \times 4.2$

$= 12.6$ cm

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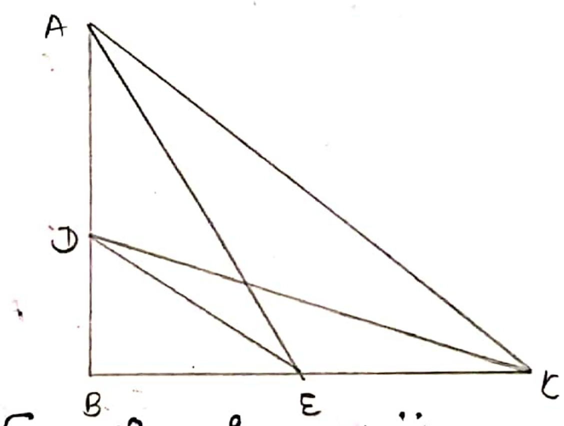
(10) ABC is right angle $\triangle$ whose $\angle B$ is right angle. If point D and E are situated on the sides AB and BC respectively, then prove that $AE^2 + CD^2 = AC^2 + DE^2$. [RBSE 2018]



Sol In $\triangle ABE$ (right $\triangle$)

$$AE^2 = AB^2 + BE^2 \quad -(i)$$

In $\triangle BCD$ (right angle $\triangle$)

$$CD^2 = BD^2 + BC^2 \quad -(ii)$$

$$AE^2 + CD^2 = AB^2 + BE^2 + BD^2 + BC^2 \quad \text{[adding (i) and (ii)]}$$

$$= AB^2 + BC^2 + BE^2 + BD^2$$

But also $AC^2 = AB^2 + BC^2 \quad -(iii)$

And $DE^2 = BD^2 + BE^2 \quad -(iv)$

$$AE^2 + CD^2 = (AB^2 + BC^2) + (BE^2 + BD^2)$$

from (iii) and (iv)

$$AE^2 + CD^2 = AC^2 + DE^2 \quad \text{HP.}$$

(11) If $\triangle ABC \sim \triangle DEF$ in which $AB = 1.6$ cm and $DE = 2.4$ cm. Find the ratio of the areas of $\triangle ABC$ and $\triangle DEF$. [RBSE 2019]

Sol.

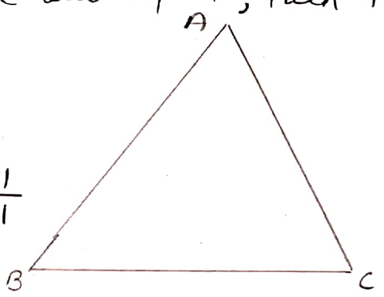
Sol Simplify the expression,

$$\frac{ar(ABC)}{ar(DEF)} = \left(\frac{AB}{DE}\right)^2$$

$$\Rightarrow \frac{ar(ABC)}{ar(DEF)} = \left(\frac{1.6}{2.4}\right)^2 = \frac{4}{9}$$

Hence, Ratio is 4 : 9.

(12) Prove that if the area of two similar triangle are equal, then they are congruent. [RBSE 2019]



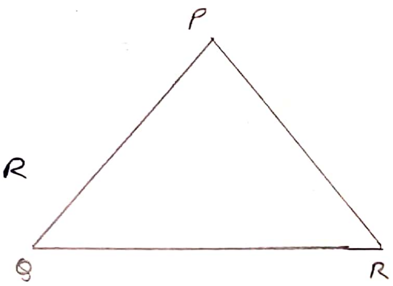
Sol. Here, $\Delta ABC \sim \Delta PQR$

Now,

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2 = \frac{1}{1}$$

Then, it can be written as,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{1}{1}$$



thus, $AB = PQ$, $BC = QR$ also, $AC = PR$

thus from SSS, they are congruent

$\Delta ABC \cong \Delta PQR$ HP.

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