RBSE BOARD

TRIGONOMETRIC IDENTITIES

PREVIOUS YEAR QUESTIONS

[2015-19]

(1) If $3\cot A = 4$, then evaluate $\frac{1-\tan^2 A}{1+\tan^2 A}$ [RBSE 2015, PART-B]

Sol. Given that,

$$3\cot A = 4$$

$$\cot A = \frac{4}{3} \quad \left(\because \cot A = \frac{B}{P}\right)$$

$$\tan A = \frac{3}{4}$$

Now, evaluate,
$$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2}$$

$$= \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}}$$

$$= \frac{7}{25} \text{ Ans.}$$

(2) Prove that $\left[\frac{1-\tan A}{1-\cot A}\right]^2 = \tan^2 A$ [RBSE 2015-PART-C]

Sol. Consider,

$$LHS = \left[\frac{1-\tan A}{1-\cot A}\right]^2 = \left[\frac{1-\frac{\sin A}{\cos A}}{1-\frac{\cos A}{\sin A}}\right]^2$$

$$= \frac{\left(\frac{\cos A - \sin A}{\cos A}\right)^2}{\left(\frac{\sin A \mp \cos A}{\sin A}\right)^2} = \frac{(\cos A - \sin A)^2}{\cos^2 A} \times \frac{\sin^2 A}{(-\sin A + \cos A)^2}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A.$$

③ Express the trigonometric ratio tan A in terms of sec A [RBSE-2016-PART-A]

Sol. Consider,

$$\tan^2 A = \sec^2 A - 1$$

$$\tan A = \sqrt{\sec A - 1}$$

④ $\frac{1+\cot^2 A}{1+\tan^2 A} = \left(\frac{1-\cot A}{1-\tan A}\right)^2$ [RBSE 2016 - PART- D]

Sol. Consider,

Taking LHS,

$$\frac{1+\cot^2 A}{1+\tan^2 A}$$

Using identities, $1+\cot^2 A = \operatorname{cosec}^2 A$

$1+\tan^2 A = \sec^2 A$

So, $$\frac{1+\cot^2 A}{1+\tan^2 A} = \frac{\operatorname{cosec}^2 A}{\sec^2 A} = \frac{\frac{1}{\sin^2 A}}{\frac{1}{\cos^2 A}} = \frac{\cos^2 A}{\sin^2 A} = \cot^2 A$$

Scanned by CamScanner

Taking RHS.

$$\left(\frac{1-\cot A}{1-\tan A}\right)^2 = \left(\frac{1-\cot A}{1-\frac{1}{\cot A}}\right)^2$$

$$= \left(\frac{1-\cot A}{\frac{\cot A-1}{\cot A}}\right)^2$$

$$= \left(\frac{(1-\cot A)\cot A}{\cot A-1}\right)^2$$

$$= (\cot A)^2\left(\frac{-(1-\cot A)}{1-\cot A}\right)^2$$

$$= \cot^2 A$$

HP.

(5) Write the trigonometric ratio of sin A in terms of cot A [RBSE - 2017 PART - A]

Sol. Simplify the expression,

We know that

$$\tan A = \frac{\sin A}{\cos A}$$

Then similarly,

$$\cot A = \frac{\cos A}{\sin A}$$

$$= \frac{\cos A}{\sin A}$$

6\. (i) Evaluate $(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)$

[RBSE-2017, PART-C]

Sol. Simplify the expression

$$(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)$$

$$=\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$$

$$=\left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$$

$$=\frac{[(\cos\theta+\sin\theta)+1][(\sin\theta+\cos\theta)-1]}{\cos\theta\cdot\sin\theta}$$

$$=\frac{(\cos\theta+\sin\theta)^2-1^2}{\cos\theta\cdot\sin\theta} \qquad *[(a+b)(a-b)=a^2-b^2]$$

$$=\frac{\cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta-1}{\sin\theta\cdot\cos\theta} \qquad *[(a+b)^2=a^2+b^2+2ab]$$

$$=\frac{1+2\sin\theta\cos\theta-1}{\cos\theta\cdot\sin\theta} \qquad *[\sin^2\theta+\cos^2\theta=1]$$

$$=\frac{2\sin\theta\cdot\cos\theta}{\cos\theta\cdot\sin\theta}=2.$$

(ii) Prove that, $\dfrac{\tan A-\sin A}{\tan A+\sin A}=\dfrac{\sec A-1}{\sec A+1}$

[RBSE-2017, PART-C]

Sol^n

LHS.

$$\frac{\tan A - \sin A}{\tan A + \sin A}$$

$$= \frac{\frac{\sin A}{\cos A} - \sin A}{\frac{\sin A}{\cos A} + \sin A}$$

$$= \frac{\sin A - \sin A \cos A / \cos A}{\sin A + \sin A \cdot \cos A / \cos A} = \frac{\sin A (1 - \cos A)}{\sin A (1 + \cos A)}$$

$$= \frac{1 - \cos A}{1 + \cos A}$$

$$= \frac{1 - \frac{1}{\sec A}}{1 + \frac{1}{\sec A}} = \frac{\frac{\sec A - 1}{\sec A}}{\frac{\sec A + 1}{\sec A}}$$

$$= \frac{\sec A - 1}{\sec A + 1}$$ HP

7 Prove that [RBSE 2018 - PART-D]

(i) $\sqrt{\frac{1 + \cos\theta}{1 - \cos\theta}} = \operatorname{cosec}\theta + \cot\theta$

Sol. Given,

LHS, $\sqrt{\frac{1 + \cos\theta}{1 - \cos\theta}}$

Rationalizing LHS

$$= \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} \times \sqrt{\frac{1+\cos\theta}{1+\cos\theta}}$$

$$= \frac{(1+\cos\theta)}{\sqrt{(1-\cos^2\theta)^{2}}} = \frac{1+\cos\theta}{\sqrt{1-\cos^2\theta}}$$

we know that

$$1-\cos^2\theta = \sin^2\theta$$

$$= \frac{1+\cos\theta}{\sqrt{\sin^2\theta}} = \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \operatorname{cosec}\theta + \cot\theta.$$

(ii) Prove

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1+\tan\theta+\cot\theta$$

Sol. Taking LHS

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1-\tan\theta}$$

$$= \frac{\tan\theta}{\frac{\tan\theta-1}{\tan\theta}} + \frac{1}{\tan\theta(1-\tan\theta)} = \frac{\tan^2\theta}{\tan\theta-1} + \frac{1}{\tan\theta(1-\tan\theta)}$$

$$= \frac{\tan^3\theta - 1^3}{\tan\theta(\tan\theta - 1)} \qquad (a^3 - b^3 = (a-b)(a^2 + ab + b^2))$$

$$= \frac{(\tan\theta - 1)(\tan^2\theta + \tan\theta + 1)}{\tan\theta(\tan\theta - 1)}$$

$$= \frac{\tan^2\theta + \tan\theta + 1}{\tan\theta}$$

$$= \tan\theta + 1 + \cot\theta \Rightarrow \text{RHS}$$

OR

(7) (i) If $\sin\theta + \cos\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, then prove that $q(p^2 - 1) = 2p$.

Sol. Given,

$$(\sin\theta + \cos\theta) = p$$

squaring both side

$$(\sin\theta + \cos\theta)^2 = p^2$$

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = p^2$$

$$p^2 = 1 + 2\sin\theta\cos\theta$$

Now, $q = \sec\theta + \operatorname{cosec}\theta$

$$= \frac{1}{\cos\theta} + \frac{1}{\sin\theta}$$

$$q = \frac{\sin\theta + \cos\theta}{\sin\theta \cdot \cos\theta}$$

Now, Putting the value of p and q in $q(p^2 - 1)$

$$= \frac{\sin\theta + \cos\theta}{\sin\theta \cdot \cos\theta}\left(1 + 2\sin\theta\cos\theta - 1\right)$$

$$= 2(\sin\theta + \cos\theta)$$

$$= 2p \qquad [\because \sin\theta + \cos\theta = p]$$

(iii) Prove that,

$$\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} = \sin A + \cos A$$

Sol. Taking LHS,

$$= \frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A}$$

$$= \frac{\cos A}{1-\frac{\sin A}{\cos A}} + \frac{\sin A}{1-\frac{\cos A}{\sin A}}$$

$$= \frac{\cos A}{\frac{\cos A - \sin A}{\cos A}} + \frac{\sin A}{\frac{\sin A - \cos A}{\sin A}}$$

$$= \frac{\sin^2 A}{\sin A - \cos A} + \frac{\cos^2 A}{(\cos A - \sin A)}$$

$$= \frac{\sin^2 A}{(\sin A - \cos A)} - \frac{\cos^2 A}{(\sin A - \cos A)} = \frac{(\sin^2 A - \cos^2 A)}{(\sin A - \cos A)}$$

$$= \frac{(\sin A - \cos A)(\sin A + \cos A)}{\sin A - \cos A}$$

$$= \sin A + \cos A$$

RHS.

Scanned by CamScanner

⑧ Prove that! [RBSE 2019 PART-D]

(i) $\dfrac{1+\sec A}{\sec A} = \dfrac{\sin^2 A}{1-\cos A}$

Sol. Consider

LHS, $\dfrac{1+\sec A}{\sec A}$

$= \dfrac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}} = \dfrac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}} = 1+\cos A \times \dfrac{1-\cos A}{1-\cos A}$

$= \dfrac{1-\cos^2 A}{1-\cos A}$

$\because [\sin^2 A + \cos^2 A = 1]$

$= \dfrac{\sin^2 A}{1-\cos A}$ RHS.

(ii) $\dfrac{\cos^2\theta}{1-\tan\theta} + \dfrac{\sin^3\theta}{\sin\theta-\cos\theta} = 1+\sin\theta\cos\theta$.

Sol. Consider,

$\dfrac{\cos^2\theta}{1-\tan\theta} + \dfrac{\sin^3\theta}{\sin\theta-\cos\theta}$

$= \dfrac{\cos^2\theta}{1-\frac{\sin\theta}{\cos\theta}} + \dfrac{\sin^3\theta}{\sin\theta-\cos\theta}$

$= \dfrac{\cos^2\theta\cdot\cos\theta}{\cos\theta-\sin\theta} + \dfrac{\sin^3\theta}{\sin\theta-\cos\theta}$

$= \dfrac{\cos^3\theta}{\cos\theta-\sin\theta} + \dfrac{\sin^3\theta}{\sin\theta-\cos\theta}$

$$= \frac{\cos^3\theta}{(\cos\theta - \sin\theta)} - \frac{\sin^3\theta}{(\cos\theta - \sin\theta)} \qquad [a^3 - b^3 = (a-b)(a^2+b^2+ab)]$$

$$= \frac{\cos^3\theta - \sin^3\theta}{(\cos\theta - \sin\theta)} = \frac{(\cos\theta - \sin\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cdot\cos\theta)}{\cos\theta - \sin\theta}$$

$$= 1 + \sin\theta\cdot\cos\theta. \Rightarrow RHS$$

HP.

OR

⑧ (i) Prove that

$$\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$$

Sol. Consider, LHS, $= \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$

$$= \frac{\sin^2\theta + (1+\cos\theta)^2}{(1+\cos\theta)\sin\theta}$$

$$= \frac{\sin^2\theta + 1 + \cos^2\theta + 2\cos\theta}{(1+\cos\theta)\sin\theta} = \frac{1+1+2\cos\theta}{(1+\cos\theta)\sin\theta}$$

$$= \frac{2(1+\cos\theta)}{(1+\cos\theta)\sin\theta}$$

$$= 2\operatorname{cosec}\theta$$

RHS.

(ii) $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

Sol. LHS. $\frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)} = \frac{\sin\theta(1-2(1-\cos^2\theta))}{\cos\theta(2\cos^2\theta-1)}$

$$= \frac{\sin\theta(1-2+2\cos^2\theta)}{\cos\theta(2\cos^2\theta-1)}$$

$$= \frac{\sin\theta}{\cos\theta}\,\frac{(2\cos^2\theta-1)}{(2\cos^2\theta-1)}$$

$$= \tan\theta \qquad RHS.$$