RBSE BOARD

CLASS - XII th

SHARMA TUTION CLASSES ▶ Let's Rule it

VECTOR ALGEBRA [PREVIOUS YEAR 2015 - 19]

(1) If the magnitude of vector $\vec{a}$ and $\vec{b}$ are 1 and 2 respectively and $\vec{a}\cdot\vec{b} = 1$. then find the angle b/w those vectors.

Sol. We know that,

$|\vec{a}| = 1, |\vec{b}| = 2$ and also $\vec{a}\cdot\vec{b} = 1$

Then, $\therefore \cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \dfrac{1}{1\times 2} = \dfrac{1}{2}$

$\cos\theta = \dfrac{1}{2} \Rightarrow \theta = 60^\circ$

(2) Find the direction cosines of x-axis.

Sol. We know that,

x-axis make an angle 0° with x-axis, 90° with y-axis and 90° with z-axis

So, $\alpha = 0^\circ$, $\beta = 90^\circ$, $\gamma = 90^\circ$

D.C are $\cos 0^\circ = l$, $\cos 90^\circ = m$, $\cos 90^\circ = n$

Thus, $l = 1$, $m = 0$, $n = 0$

③ If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is $\perp$ to vector $\vec{c}$, then find the value of λ. [RBSE 2015]

Sol. Simplify the expression,

The given vectors are
$$\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$$
$$\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$$
$$\vec{c} = 3\hat{i} + \hat{j}$$

$$(\vec{a} + \lambda\vec{b}) \cdot c = 0$$

$$[(2+\lambda)\hat{i} + (2-2\lambda)\hat{j} + (3+\lambda)\hat{k}] \cdot (3\hat{i} + \hat{j}) = 0$$

$$(2+\lambda)\hat{i} \cdot 3\hat{i} + (2-2\lambda)\hat{j} \cdot \hat{j} = 0$$

$$6 + 3\lambda + 2 - 2\lambda = 0$$

$$\boxed{\lambda = -8}$$

④ If $a = 2\hat{i} - \hat{j} + 5\hat{k}$ and $b = 4\hat{i} - 2\hat{j} + \lambda k$ such that $a \parallel b$, find the value of λ. [RBSE 2016]

Sol Here, $a \parallel b$

then, $a = Kb$

$$(2\hat{i} - \hat{j} + 5\hat{k}) = K(4\hat{i} - 2\hat{j} + \lambda k)$$

Now, comparing both

$2 = 4k \Rightarrow k = \frac{1}{2}$

$-1 = -2$

$5k = \lambda k$ and $\lambda = \frac{k}{5} = \frac{1}{2 \times 5} = \frac{1}{10}$

(5) Find the direction cosine of the line $\frac{x}{4} = \frac{y}{7} = \frac{z}{4}$. [RBSE 2016]

Sol. Here,

Direction ratio of the line are $\frac{x}{4} = \frac{y}{7} = \frac{z}{4}$

Also, direction cosine of the line are $\frac{x}{4} = \frac{y}{7} = \frac{z}{4}$ are

$$\frac{4}{\sqrt{4^2+7^2+4^2}}, \frac{7}{\sqrt{4^2+7^2+4^2}}, \frac{4}{\sqrt{4^2+7^2+4^2}}$$

$$\frac{4}{\sqrt{81}}, \frac{7}{\sqrt{81}}, \frac{4}{\sqrt{81}} \Rightarrow \frac{4}{9}, \frac{7}{9}, \frac{4}{9} \text{ are DC.}$$

(6) If a, b and c are unit vector such that $a+b+c = 0$, find the value of $a \cdot b + b \cdot c + c \cdot a$. [RBSE 2016]

Sol Here, Given, $a+b+c = 0$

Squaring both side.

$$(\vec{a}+\vec{b}+\vec{c})^2 = 0$$

$$(\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b}+\vec{c}) = 0$$

$$|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+ 2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = 0$$

Given, $|\vec{a}| = 1$, $|\vec{b}| = 1$, $|\vec{c}| = 1$

So, $1+1+1+ 2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = 0$

$$2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = -3$$

$$\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a} = \frac{-3}{2}$$

(7) Find a unit vector ⊥ to each of the vector $2a+b$ and $2a-2b$, where $a = \hat{i}+2\hat{j}-\hat{k}$ $\vec{b} = \hat{i}+\hat{j}+\hat{k}$. [RBSE 2016]

Sol Here, given $\vec{a} = \hat{i}+2\hat{j}-\hat{k}$

$\vec{b} = \hat{i}+\hat{j}+\hat{k}$

$$2\vec{a}+\vec{b} = 2(\hat{i}+2\hat{j}-\hat{k})+\hat{i}+\hat{j}+\hat{k} = 3\hat{i}+5\hat{j}-\hat{k}$$

$$\vec{a}-2\vec{b} = (\hat{i}+2\hat{j}-\hat{k})-2(\hat{i}+\hat{j}+\hat{k}) = \hat{i}-3\hat{k}$$

Then, the vector ⊥ to both $2\vec{a}+\vec{b}$ and $\vec{a}-2\vec{b}$ is $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & -1 \\ -1 & 0 & -3 \end{vmatrix}$

$\hat{i}(-15-0)-\hat{j}(-9-1)+\hat{k}(3\cdot0+5)$

$-15\hat{i}+10\hat{j}+5\hat{k}$

Unit vector is given by $\dfrac{[-15\hat{i}+10\hat{j}+5\hat{k}]}{\sqrt{(15)^2+(10)^2+(5)^2}} = \dfrac{-15\hat{i}+10\hat{j}+5\hat{k}}{\sqrt{225+100+25}}$

$$= \frac{-15\hat{i}+10\hat{j}+5\hat{k}}{\sqrt{350}}$$

$$= \frac{-15\hat{i}+10\hat{j}+5\hat{k}}{5\sqrt{14}}$$

$$= \frac{-3\hat{i}+2\hat{j}+\hat{k}}{\sqrt{14}}.$$

(8) If vector $a = 2\hat{i}-2\hat{j}+2\hat{k}$ and vector $b = \hat{i}+\hat{j}-\hat{k}$, then find the unit vector along the vector $(a+b)$. [RBSE 2017]

Sol. Here, $\vec{a} = 2\hat{i}-2\hat{j}+2\hat{k}$

$\vec{b} = \hat{i}+\hat{j}-\hat{k}$

$\vec{a}+\vec{b} = 3\hat{i}-\hat{j}+\hat{k}$

Unit vector along $\vec{a}+\vec{b} = \dfrac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|} = \dfrac{3\hat{i}-\hat{j}+\hat{k}}{\sqrt{3^2+(-1)^2+(1)^2}} = \dfrac{3\hat{i}-\hat{j}+\hat{k}}{\sqrt{11}}$

⑨ If a line makes 120°, 45° and 90° angles, with the x, y and z-axis respectively then find direction cosines. [RBSE 2017]

Sol. If a line makes α, β and γ with the x, y and z-axis respectively, then the direction cosines are given by

$$l = \cos\alpha$$

$$m = \cos\beta$$

$$n = \cos\gamma$$

So, $l = \cos 120^\circ = \frac{-1}{2}$

$m = \cos 45^\circ = \frac{-1}{\sqrt{2}}$

$n = \cos 90^\circ = 0$

⑩ If $a = 5\hat{i} - \hat{j} - 3\hat{k}$ and $b = \hat{i} - 3\hat{j} - 5k$, then find the angle b/w the vector $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ [RBSE 2017]

Sol. Here, $\vec{a} + \vec{b} = 6\hat{i} - 4\hat{j} - 8\hat{k}$

also $\vec{a} - \vec{b} = 4\hat{i} - 4\hat{j} - 2\hat{k}$

Now, Angle b/w both can be given by

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$$\cos\theta = \frac{(a+b)(a-b)}{|a+b||a-b|} = \frac{(6\hat{i}+2\hat{j}-8k)\cdot(4\hat{i}-4\hat{j}-2\hat{k})}{\sqrt{6^2+2^2+(-8)^2}\sqrt{(4)^2+(-4)^2+(-2)^2}}$$

$$\cos\theta = \frac{24-8-16}{\sqrt{104}\sqrt{36}}$$

$$\cos\theta = 0 \Rightarrow \boxed{\theta = 90^\circ}$$

(11) Find the area of llgm whose adjacent sides are vector $a = \hat{i}-\hat{j}+3\hat{k}$ and $\vec{b} = 2\hat{i}-7\hat{j}-\hat{k}$. [RBSE 2017]

Sol Here, $\vec{a} = \hat{i}-\hat{j}+3\hat{k}$

$\vec{b} = 2\hat{i}-7\hat{j}+\hat{k}$

$$A = |\vec{a}\times\vec{b}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$$

$$= \hat{i}(-1+21) - \hat{j}(1-6) + k(-7+2)$$

$$= 20\hat{i} + 5\hat{j} - 5\hat{k}$$

$$A = |\vec{a}\times\vec{b}| = \sqrt{(20)^2+(5)^2+(-5)^2} = \sqrt{400+25+25} = \sqrt{450}$$

(12) Find a vector of magnitude 5 units along the vector $\hat{i}-2\hat{j}+2\hat{k}$. [RBSE 2018]

Sol. Here, let $a = \hat{i}-2\hat{j}+2\hat{k}$

Now, vector $\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i}-2\hat{j}+2\hat{k}}{\sqrt{(2)^2+(2)^2+(1)^2}} = \frac{\hat{i}-2\hat{j}+2\hat{k}}{\sqrt{9}} = \frac{\hat{i}-2\hat{j}+2\hat{k}}{3}$

Now, vector of magnitude 5 units can be found by $a = 5 \times \vec{a}$

$= 5 \times \frac{(\hat{i}-2\hat{j}+2\hat{k})}{3}$

$= \frac{5}{3}(\hat{i}-2\hat{j}+2\hat{k})$

(13) Find the projection of the vector $\hat{i}-\hat{j}$ on the vector $\hat{i}+\hat{j}$. [RBSE 2018]

Sol. Given two vector, $\hat{i}-\hat{j}$ and $\hat{i}+\hat{j}$

let $\vec{a} = \hat{i}-\hat{j}$, $\vec{b} = \hat{i}+\hat{j}$,

Now, Projection of $\vec{a}$ on $\vec{b}$ is given by $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

Then, $\frac{(\hat{i}-\hat{j})(\hat{i}+\hat{j})}{\sqrt{1^2+1^2}} = \frac{1-1}{\sqrt{2}} = 0$

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(14) For any vector a, prove that $|a \times \hat{i}|^2 + |a \times \hat{j}|^2 + |a \times \hat{k}|^2 = 2|a|^2$ [RBSE 2018]

Sol. Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ also $|\vec{a}| = \sqrt{x^2+y^2+z^2}$

then $|a \times \hat{i}|^2 + |a \times \hat{j}|^2 + |a \times \hat{k}|^2$

$= |(x\hat{i}+y\hat{j}+z\hat{k}) \times \hat{i}|^2 + |(x\hat{i}+y\hat{j}+z\hat{k}) \times \hat{j}|^2 + |(x\hat{i}+y\hat{j}+z\hat{k}) \times \hat{k}|^2$

$= |(-y\hat{k}+z\hat{k})|^2 + |(x\hat{k}-z\hat{i})|^2 + |(-x\hat{j}+y\hat{i})|^2$

$= \left(\sqrt{(-y)^2+(z)^2}\right)^2 + \left(\sqrt{x^2+(-z)^2}\right)^2 + \left(\sqrt{(-x)^2+(y)^2}\right)^2$

$= y^2+z^2+x^2+z^2+x^2+y^2$

$= 2(x^2+y^2+z^2)$

$= 2\left(\sqrt{x^2+y^2+z^2}\right)^2$

$= 2|\vec{a}|^2$

OR

(15) For any vector a, prove that $a = (a\cdot\hat{i})\cdot\hat{i} + (a\cdot\hat{j})\cdot\hat{j} + (a\cdot\hat{k})\cdot\hat{k}$ [RBSE 2018]

Sol. Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

$|\vec{a}| = \sqrt{x^2+y^2+z^2}$

Now, $a = ((x\hat{i}+y\hat{j}+z\hat{k})\cdot\hat{i})\hat{i} + ((x\hat{i}+y\hat{j}+z\hat{k})\cdot\hat{j})\hat{j} + ((x\hat{i}+y\hat{j}+z\hat{k})\cdot\hat{k})\hat{k}$

$= x\hat{i} + y\hat{j} + z\hat{k}$

$a = a$ HP.

(16) Find the angle b/w $2\hat{i}-\hat{j}$ and $\hat{i}+2\hat{j}$.

Sol. Vector $a = 2\hat{i}-\hat{j}$

$b = \hat{i}+2\hat{j}$

we know that,

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{(2\hat{i}-\hat{j})(\hat{i}+2\hat{j})}{\sqrt{4+1}\sqrt{4+1}}$$

$$\cos\theta = \frac{2-2}{5} \Rightarrow \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$$

(17) If $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a}\cdot\vec{b} = 12$. then find the value of $\sin\theta$ where θ is the angle b/w the vector a and b.

Sol. Here, given $\vec{a}\cdot\vec{b} = 12$, $|\vec{a}| = 10$, $|\vec{b}| = 2$

So, $$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{12}{10\times 2} = \frac{3}{5}$$

$\cos\theta = \frac{3}{5}$, $\sin^2\theta = 1 - \cos^2\theta$

$$\sin^2\theta = 1 - \left(\frac{3}{5}\right)^2 = 1 - \frac{9}{25} = \frac{16}{25}$$

$$\sin\theta = \frac{4}{5}.$$

(18) If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, then prove that $(a-d)$ is $\parallel$ to $(b-c)$

Sol Here,

$$\vec{a} \times \vec{b} = \vec{c} \times \vec{d} \quad -(i)$$

$$\vec{a} \times \vec{c} = \vec{b} \times \vec{d} \quad -(ii)$$

Subtracting (ii) from (i)

$$(\vec{a} \times \vec{b}) - (\vec{a} \times \vec{c}) = (\vec{c} \times \vec{d}) - (\vec{b} \times \vec{d})$$

$$\vec{a} \times (\vec{b} - \vec{c}) = (\vec{c} - \vec{b}) \times \vec{d}$$

$$\vec{a} \times (\vec{b} - \vec{c}) - (\vec{c} - \vec{b}) \times \vec{d} = 0 \Rightarrow \vec{a} \times (\vec{b} - \vec{c}) - \vec{d} \times (\vec{b} - \vec{c}) = 0$$

$$(\vec{b} - \vec{c})(\vec{a} - \vec{d}) = 0$$

HP.